

A soft origin for CKM-type CP violation

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January 11, 2013

Abstract

We present a two-Higgs-doublet model, with a \mathbb{Z}_3 symmetry, in which CP violation originates solely in a soft (dimension-2) coupling in the scalar potential, and reveals itself solely in the CKM (quark mixing) matrix. In particular, in the mass basis the Yukawa interactions of the neutral scalars are all real. The model has only eleven parameters to fit the six quark masses and the four independent CKM-matrix observables. We find regions of parameter space in which the flavour-changing neutral couplings are so suppressed that they allow the scalars to be no heavier than a few hundred GeV.

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1 Introduction and notation

One of the conceptually simplest extensions of the Standard Model (SM) of the electroweak interactions consists in allowing for $n_H > 1$ gauge-SU(2) “Higgs” doublets. In such multi-Higgs-doublet models (MHDMs) CP violation may occur in various places: in the quark mixing matrix (CKM matrix) just as in the SM, in the Yukawa couplings of the scalars to the quarks,¹ in the mixing of the scalars (in particular, scalar–pseudoscalar mixing), or in the self-interactions (cubic and quartic interactions) among the scalars. Unfortunately, MHDMs in general lead to the existence of flavour-changing neutral currents (FCNC),² which are severely restricted by the experimental data.

The simplest MHDMs are, of course, two-Higgs-doublet models (2HDMs) [1], which have lately been the object of intense scrutiny [2]. The Yukawa interactions of the quarks in the 2HDM are written

$$\mathcal{L}_{\text{Yuk}} = -\bar{Q}_L \sum_{k=1}^2 \left(\phi_k \Gamma_k n_R + \tilde{\phi}_k \Delta_k p_R \right) + \text{H.c.}, \quad (1)$$

where $\phi_{1,2}$ are the two scalar gauge-SU(2) doublets, $\tilde{\phi}_k \equiv i\tau_2 \phi_k^*$ for $k = 1, 2$, Γ_k and Δ_k are (in general, complex) 3×3 matrices in flavour space, and Q_L , n_R , and p_R denote the 3-vectors (in flavour space) of quark left-handed doublets, right-handed charge $-1/3$ quarks, and right-handed charge $+2/3$ quarks, respectively. In order for the U(1) gauge group of electromagnetism to be preserved, the Higgs doublets are assumed to have vacuum expectation values (VEVs) of the form

$$\langle 0 | \phi_k | 0 \rangle = \begin{pmatrix} 0 \\ v_k e^{i\theta_k} \end{pmatrix}, \quad \langle 0 | \tilde{\phi}_k | 0 \rangle = \begin{pmatrix} v_k e^{-i\theta_k} \\ 0 \end{pmatrix}, \quad (2)$$

with real and non-negative v_k . The quark mass matrices are then

$$M_n = \sum_{k=1}^2 v_k e^{i\theta_k} \Gamma_k, \quad (3)$$

$$M_p = \sum_{k=1}^2 v_k e^{-i\theta_k} \Delta_k. \quad (4)$$

¹In this paper we neglect the lepton sector.

²More precisely, quark-flavour-changing Yukawa interactions of the neutral scalars.

These are bi-diagonalized as usual by unitary matrices $U_{L,R}^{n,p}$,

$$U_L^{n\dagger} M_n U_R^n = M_d = \text{diag}(m_d, m_s, m_b), \quad (5)$$

$$U_L^{p\dagger} M_p U_R^p = M_u = \text{diag}(m_u, m_c, m_t), \quad (6)$$

and the CKM matrix is $V = U_L^{p\dagger} U_L^n$. The quantity $v = \sqrt{v_1^2 + v_2^2} = (2\sqrt{2}G_F)^{-1/2} \approx 174 \text{ GeV}$ is responsible for the masses of the W^\pm and Z^0 gauge bosons. It is convenient to use the ‘Higgs basis’,

$$H_1 = (v_1 e^{-i\theta_1} \phi_1 + v_2 e^{-i\theta_2} \phi_2) / v \quad (7)$$

$$= \begin{pmatrix} G^+ \\ v + (h + iG^0) / \sqrt{2} \end{pmatrix}, \quad (8)$$

$$H_2 = (v_2 e^{-i\theta_1} \phi_1 - v_1 e^{-i\theta_2} \phi_2) / v \quad (9)$$

$$= \begin{pmatrix} C^+ \\ (H + iA) / \sqrt{2} \end{pmatrix}, \quad (10)$$

in which only H_1 has VEV, which is precisely v . The fields G^+ and G^0 are the would-be Goldstone bosons. The field C^+ is a physical charged scalar. The neutral fields h , H , and A in general mix to form the three physical neutral scalars of the 2HDM. We define the matrices

$$N_n = v_2 e^{i\theta_1} \Gamma_1 - v_1 e^{i\theta_2} \Gamma_2, \quad (11)$$

$$N_p = v_2 e^{-i\theta_1} \Delta_1 - v_1 e^{-i\theta_2} \Delta_2, \quad (12)$$

and

$$N_d = U_L^{n\dagger} N_n U_R^n, \quad (13)$$

$$N_u = U_L^{p\dagger} N_p U_R^p. \quad (14)$$

Equation (1) then becomes

$$\begin{aligned} \mathcal{L}_{\text{Yuk}} = & -\bar{d}_L M_d d_R - \bar{u}_L M_u u_R \\ & - \frac{h}{\sqrt{2}v} (\bar{d}_L M_d d_R + \bar{u}_L M_u u_R) - \frac{H}{\sqrt{2}v} (\bar{d}_L N_d d_R + \bar{u}_L N_u u_R) \\ & - \frac{iG^0}{\sqrt{2}v} (\bar{d}_L M_d d_R - \bar{u}_L M_u u_R) - \frac{iA}{\sqrt{2}v} (\bar{d}_L N_d d_R - \bar{u}_L N_u u_R) \\ & + \frac{G^+}{v} \bar{u} (M_u V \gamma_L - V M_d \gamma_R) d + \frac{C^+}{v} \bar{u} (N_u^\dagger V \gamma_L - V N_d \gamma_R) d \\ & + \text{H.c.}, \end{aligned} \quad (15)$$

where d and u denote the column vectors in flavour space of the charge $-1/3$ and charge $+2/3$ quarks, respectively, in the mass basis, and $\gamma_{L,R}$ are the chirality projection matrices in Dirac space. Since the matrices N_d and N_u are not necessarily diagonal, the terms $\bar{d}_L N_d d_R$ and $\bar{u}_L N_u u_R$ in general include potentially problematic FCNC.

We spot in the Yukawa interactions of equation (15) three possible manifestations of CP violation:

The CKM matrix V may contain a complex phase, just as in the SM.

The matrices N_d and N_u may be complex.

The scalars h and H may mix with the pseudoscalar A .³

One further manifestation of CP violation may occur in the cubic and quartic interactions among the scalars. It is the purpose of this paper to present a 2HDM with an additional symmetry such that only the first one of the above four manifestations of CP violation occurs; namely, the matrices N_d and N_u are real, the scalars do not mix with the pseudoscalar, and the cubic and quartic interactions among the (neutral and charged) scalars respect CP invariance. Additionally, our model shows that the FCNC may be quite suppressed even when all the scalars have relatively low (less than 1 TeV) masses.

2 The model: Yukawa couplings

Our model is a 2HDM supplemented by a particular \mathbb{Z}_3 symmetry and by the usual CP symmetry. Let $\omega = \exp(2i\pi/3)$. Then, under the \mathbb{Z}_3 symmetry, the following matter fields transform as

$$\begin{aligned}\phi_2 &\rightarrow \omega^2 \phi_2, \\ Q_{L1} &\rightarrow \omega^2 Q_{L1}, \quad Q_{L2} \rightarrow \omega Q_{L2}, \\ n_{R3} &\rightarrow \omega n_{R3}, \quad p_{R3} \rightarrow \omega p_{R3},\end{aligned}\tag{16}$$

³If this mixing exists, *i.e.* if the three physical neutral scalars are mixtures of all three h , H , and A , then A is not a physical particle and it does not make sense to separate the physical neutral scalars into two scalars and one pseudoscalar.

and all other fields remain invariant. This symmetry forces the Yukawa-coupling matrices to have the following form [3]:

$$\Gamma_1, \Delta_1 \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ \times & \times & 0 \end{pmatrix}, \quad \Gamma_2 \sim \begin{pmatrix} \times & \times & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}, \quad \Delta_2 \sim \begin{pmatrix} 0 & 0 & \times \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

where the symbol \times denotes a non-zero matrix entry. The standard CP symmetry forces all those non-zero entries of the Yukawa-coupling matrices to be real. Therefore, the mass matrices end up being

$$M_n = e^{i\theta_1} \begin{pmatrix} e^{i\theta} x & 0 & 0 \\ 0 & 0 & a \\ b & c & e^{i\theta} y \end{pmatrix}, \quad (17)$$

$$M_p = e^{-i\theta_1} \begin{pmatrix} 0 & 0 & e^{-i\theta} a' \\ e^{-i\theta} b' & e^{-i\theta} c' & x' \\ y' & 0 & 0 \end{pmatrix}, \quad (18)$$

where $\theta = \theta_2 - \theta_1$ and a, b, c, x, \dots , and y' are real. In equation (17) we have already assumed a rotation between n_{R1} and n_{R2} which renders zero the (1,2) entry of Γ_2 ; in the same way, in equation (18) a rotation between p_{R1} and p_{R2} has been used to make $(\Delta_1)_{32} = 0$. The matrices parameterizing the Yukawa couplings of H_2 are

$$N_n = e^{i\theta_1} \begin{pmatrix} -e^{i\theta} x/r & 0 & 0 \\ 0 & 0 & ra \\ rb & rc & -e^{i\theta} y/r \end{pmatrix}, \quad (19)$$

$$N_p = e^{-i\theta_1} \begin{pmatrix} 0 & 0 & -e^{-i\theta} a'/r \\ -e^{-i\theta} b'/r & -e^{-i\theta} c'/r & rx' \\ ry' & 0 & 0 \end{pmatrix}, \quad (20)$$

where $r = v_2/v_1$.

Let

$$O_L^{nT} \begin{pmatrix} |x| & 0 & 0 \\ 0 & 0 & |a| \\ |b| & |c| & |y| \end{pmatrix} O_R^n = M_d, \quad (21)$$

$$O_L^{pT} \begin{pmatrix} 0 & 0 & |a'| \\ |b'| & |c'| & |x'| \\ |y'| & 0 & 0 \end{pmatrix} O_R^p = M_u, \quad (22)$$

where $O_{L,R}^{n,p}$ are real orthogonal matrices. It is then clear that

$$U_L^{n\dagger} = O_L^{nT} \text{diag} \left(1, \frac{abxy}{|abxy|} e^{2i\theta}, \frac{bx}{|bx|} e^{i\theta} \right), \quad (23)$$

$$U_R^n = e^{-i\theta_1} \text{diag} \left(\frac{x}{|x|} e^{-i\theta}, \frac{bcx}{|bcx|} e^{-i\theta}, \frac{bxy}{|bxy|} e^{-2i\theta} \right) O_R^n, \quad (24)$$

$$U_L^{p\dagger} = O_L^{pT} \text{diag} \left(1, \frac{a'x'}{|a'x'|} e^{-i\theta}, \frac{a'b'x'y'}{|a'b'x'y'|} e^{-2i\theta} \right), \quad (25)$$

$$U_R^p = e^{i\theta_1} \text{diag} \left(\frac{a'b'x'}{|a'b'x'|} e^{2i\theta}, \frac{a'c'x'}{|a'c'x'|} e^{2i\theta}, \frac{a'}{|a'|} e^{i\theta} \right) O_R^p. \quad (26)$$

The CKM matrix is

$$V = O_L^{pT} \text{diag} (1, e^{i\alpha}, \pm e^{i\alpha}) O_L^n, \quad (27)$$

where

$$e^{i\alpha} = \frac{a'x'aybx}{|a'x'aybx|} e^{-3i\theta}, \quad \pm e^{i\alpha} = \frac{a'x'b'y'bx}{|a'x'b'y'bx|} e^{-3i\theta}. \quad (28)$$

One sees that *the complexity of the CKM matrix originates exclusively from the phase 3θ* , which is the only phase with physical consequences in our model. One easily finds the matrices parametrizing the non-diagonal Yukawa couplings:

$$N_d = O_L^{nT} \begin{pmatrix} -|x|/r & 0 & 0 \\ 0 & 0 & r|a| \\ r|b| & r|c| & -|y|/r \end{pmatrix} O_R^n, \quad (29)$$

$$N_u = O_L^{pT} \begin{pmatrix} 0 & 0 & -|a'|/r \\ -|b'|/r & -|c'|/r & r|x'| \\ r|y'| & 0 & 0 \end{pmatrix} O_R^p. \quad (30)$$

These matrices are real. Thus, *in our model there is no CP violation from the FCNC matrices.*

3 The model: scalar potential

The scalar potential of our model is

$$V = V_{\text{sym}} + V_{\text{SB}}, \quad (31)$$

$$V_{\text{sym}} = \mu_1 \phi_1^\dagger \phi_1 + \mu_2 \phi_2^\dagger \phi_2 + \frac{\lambda_1}{2} (\phi_1^\dagger \phi_1)^2 + \frac{\lambda_2}{2} (\phi_2^\dagger \phi_2)^2 + \lambda_3 \phi_1^\dagger \phi_1 \phi_2^\dagger \phi_2 + \lambda_4 \phi_1^\dagger \phi_2 \phi_2^\dagger \phi_1, \quad (32)$$

$$V_{\text{SB}} = -|\mu_3| \left(e^{-i\vartheta} \phi_1^\dagger \phi_2 + e^{i\vartheta} \phi_2^\dagger \phi_1 \right), \quad (33)$$

where V_{sym} respects the \mathbb{Z}_3 and CP symmetries of the model while V_{SB} breaks both those symmetries, but only softly. The soft-breaking term is unique and is as general as possible. Note that V_{sym} coincides with the Peccei–Quinn potential [4]. The minimization of the potential leads to the vacuum phase θ being equal to the phase ϑ in V_{SB} . Thus, in our model *the origin of CP violation lies exclusively in a soft term in the scalar potential*.⁴

The equations for vacuum stability read, besides $\theta = \vartheta$,

$$\mu_1 = |\mu_3| \frac{v_2}{v_1} - \lambda_1 v_1^2 - (\lambda_3 + \lambda_4) v_2^2, \quad (34)$$

$$\mu_2 = |\mu_3| \frac{v_1}{v_2} - \lambda_2 v_2^2 - (\lambda_3 + \lambda_4) v_1^2. \quad (35)$$

If we define

$$m_A^2 = \frac{|\mu_3| v^2}{v_1 v_2}, \quad m_C^2 = m_A^2 - \lambda_4 v^2, \quad (36)$$

⁴ Note that ours is a model with soft CP breaking—the Lagrangian does not enjoy CP symmetry because of the presence of the μ_3 term. This is distinct from a model [5] in which spontaneous CP violation is achieved through the addition to the Lagrangian of a soft (dimension-2) term which breaks some other internal symmetry but does not break CP. Spontaneous CP violation usually leads to CP violation in the scalar sector, in particular through scalar–pseudoscalar mixing. However, recently a model was found [6] in which there is spontaneous CP violation but the scalar sector still preserves CP.

then we easily find that the part of V which is bilinear in the fields is

$$\begin{aligned}
V_{\text{bilinear}} = & \frac{m_A^2}{2} (A^2 + H^2) + m_C^2 C^- C^+ \\
& + \frac{\lambda_1 v_1^4 + \lambda_2 v_2^4 + 2(\lambda_3 + \lambda_4) v_1^2 v_2^2}{v^2} h^2 \\
& + [\lambda_1 + \lambda_2 - 2(\lambda_3 + \lambda_4)] \frac{v_1^2 v_2^2}{v^2} H^2 \\
& + 2v_1 v_2 \frac{\lambda_1 v_1^2 - \lambda_2 v_2^2 + (\lambda_3 + \lambda_4)(v_2^2 - v_1^2)}{v^2} hH. \quad (37)
\end{aligned}$$

One sees that A does not mix with h and H . *In our model there is no scalar–pseudoscalar mixing.*

Moreover, *in our model there is no CP violation in the self-interactions of the scalars.* This follows from the fact that in a general 2HDM there is only one gauge-invariant vacuum phase— θ —and in our specific 2HDM there are only two terms in the scalar potential—those with coefficient $|\mu_3| \exp(\pm i\vartheta)$ —which are sensitive to that phase. The vacuum phase adjusts in such a way as to offset the phase of those terms in the scalar potential so that the final potential has no phase at all.

4 The fit: procedure

4.1 First stage

As seen in equations (21) and (22), the six quark masses depend only on ten parameters: $|a|$, $|b|$, $|c|$, $|x|$, $|y|$, $|a'|$, $|b'|$, $|c'|$, $|x'|$, and $|y'|$. Then, from equation (27), the CKM matrix V , which contains four independent observables, depends on one additional parameter, the phase θ .⁵ One thus has to fit ten observables by means of eleven parameters.⁶

We have assumed throughout that the contributions to quark decays from tree-level diagrams with intermediate scalars are much smaller than the contributions from diagrams with intermediate W^\pm . We thus assume that the

⁵The CKM matrix additionally depends on the signs of $a'x'bxay$ and of $a'x'bxby'$, as seen in equation (28).

⁶Even when the number of parameters is larger than the number of observables to be fitted, obtaining a good fit is not always possible. The fact that our model passes this test is interesting in itself.

SM extractions of $|V_{us}|$, $|V_{cb}|$, and $|V_{ub}|$ still hold in our model. These three CKM-matrix elements and the quark masses are allowed to take any value within their Particle Data Group (PDG) allowed ranges [7]. In our fits $|V_{td}|$ is left free, but we have found that, once the various experimental constraints to be discussed below are included, a good fit is obtained only when $|V_{td}|$ lies roughly in the SM-allowed range.

We then proceed to analyze the FCNC of our model. These are governed by the matrices N_d and N_u in equations (29) and (30), respectively. Those matrices involve the extra parameter $r = v_2/v_1$.

In our analysis of the FCNC, we consider only their contributions to the mixing in the neutral-meson–antimeson systems K , B_d , B_s , and D . The relevant quantity is the off-diagonal matrix element M_{12} connecting each meson to the corresponding antimeson. That matrix element receives contributions both from an SM box diagram and a tree-level diagram involving the FCNC. We denote the latter by NP (for “New Physics”) and write

$$M_{12} = M_{12}^{\text{SM}} + M_{12}^{\text{NP}}. \quad (38)$$

In order to shorten our text we shall follow the notation in the textbook [8] and freely use its equations with the prefix BLS. For the K system, M_{12}^{SM} and the quantities relevant for its determination can be found in equations (BLS-17.14), (BLS-17.16), (BLS-B.15), (BLS-B.16), and (BLS-13.50); the expressions for the other neutral-meson systems are obtained by straightforward modifications of the quarks and mesons involved. The quark masses and CKM-matrix elements utilized in the calculation of M_{12} are those produced by each of our fits; in addition, we use some other quantities shown in Appendix A.

The calculation of M_{12}^{NP} is in equation (BLS-22.76).⁷ This calculation requires equations (BLS-22.29), (BLS-22.33), and (BLS-22.73). Since there is no scalar–pseudoscalar mixing in our model, one has

$$M_{12}^{\text{NP}} = M_{12}^A + M_{12}^{Hh}, \quad (39)$$

where

M_{12}^A originates in the tree-level exchange of the pseudoscalar (parity-odd) A ;

⁷That equation contains a sign mistake in the hadronic matrix elements in the vacuum-insertion approximation, which we have corrected.

M_{12}^{Hh} originates in the tree-level exchange of the two physical parity-even scalars S_1 and S_2 , with masses m_1 and m_2 , respectively.

The scalars are mixtures of H and h through

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos \psi & \sin \psi \\ -\sin \psi & \cos \psi \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \end{pmatrix}. \quad (40)$$

One defines an effective mass m_{eff} in the scalar sector:

$$\frac{1}{m_{\text{eff}}^2} = \frac{\sin^2 \psi}{m_1^2} + \frac{\cos^2 \psi}{m_2^2}. \quad (41)$$

One then has, for the K system,

$$\begin{aligned} M_{12}^A &= \frac{f_K^2 m_K}{192 v^2} \frac{1}{m_A^2} \left\{ - \left[1 + \frac{m_K^2}{(m_s + m_d)^2} \right] [(N_d)_{21}^* - (N_d)_{12}]^2 \right. \\ &\quad \left. + \left[1 + \frac{11 m_K^2}{(m_s + m_d)^2} \right] [(N_d)_{21}^* + (N_d)_{12}]^2 \right\}, \end{aligned} \quad (42)$$

$$\begin{aligned} M_{12}^{Hh} &= \frac{f_K^2 m_K}{192 v^2} \frac{1}{m_{\text{eff}}^2} \left\{ \left[1 + \frac{m_K^2}{(m_s + m_d)^2} \right] [(N_d)_{21}^* + (N_d)_{12}]^2 \right. \\ &\quad \left. - \left[1 + \frac{11 m_K^2}{(m_s + m_d)^2} \right] [(N_d)_{21}^* - (N_d)_{12}]^2 \right\}. \end{aligned} \quad (43)$$

Both m_K and f_K are given in Appendix A. In equations (42) and (43), we should note that the matrix N_d is real in our model, therefore both M_{12}^A and M_{12}^{Hh} are real.

In the K system, we use M_{12} to fit

$$\Delta m_K = 2 |M_{12}|, \quad (44)$$

$$e^{-i\pi/4} \epsilon_K = - \frac{\text{Im}(M_{12} \lambda_u^2)}{\sqrt{2} \Delta m_K |\lambda_u|^2}, \quad (45)$$

where $\lambda_u = V_{us}^* V_{ud}$. In the K system there are important long-distance contributions to M_{12} , which we do not know how to compute precisely. Therefore, in that system we use for M_{12}^{SM} only the short-distance box diagrams, but allow Δm_K calculated by using equations (38) and (44) to be in between one half and twice the experimental value.

In the B_d and B_s systems⁸ we fit Δm_{B_d} and Δm_{B_s} by using a formula analogous to equation (44).

There are uncertainties in the “bag parameters” used in M_{12}^{SM} . In M_{12}^{NP} , we use the vacuum-insertion approximation to calculate the values of the hadronic matrix elements, and do not allow for corrections to the matrix elements provided by that approximation. In order to allow for these theoretical uncertainties, we let our results for ϵ_K , Δm_{B_d} , and Δm_{B_s} differ from the experimental values by at most 10%.

We fit two more quantities, $\sin(2\beta)$ and $\sin(2\alpha)$. These are computed in the following way. For the K , B_d , and B_s decays we define⁹

$$\frac{q}{p} = \frac{M_{12}^*}{|M_{12}|}. \quad (46)$$

CP violation in $B_d \rightarrow \psi K_S$ is determined by

$$\lambda_{\psi K_S} = \left(\frac{q}{p}\right)_{B_d} \frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}} \left(\frac{p}{q}\right)_K. \quad (47)$$

By using equations (BLS-28.24), (BLS-30.34), and (BLS-30.35), we know that in the SM $\lambda_{\psi K_S} = \exp(-2i\beta)$, where β is a certain phase of the CKM matrix.¹⁰ We therefore use

$$\sin(2\beta) = -\text{Im } \lambda_{\psi K_S} \quad (48)$$

and compare our $-\text{Im } \lambda_{\psi K_S}$ to the current experimental value of $\sin(2\beta)$. In this way we constrain the NP contributions to M_{12} in both the B_d and K systems, through equations (46) and (47).

An isospin analysis of the decays $B_d \rightarrow \pi\pi$ may be used, together with the analysis of $B_d \rightarrow \rho\pi$ and $B_d \rightarrow \rho\rho$, to extract

$$\lambda_{\pi\pi} = \left(\frac{q}{p}\right)_{B_d} \frac{V_{ub}V_{ud}^*}{V_{ub}^*V_{ud}}. \quad (49)$$

Using equations (BLS-28.24) and (BLS-30.35), we see that in the SM $\lambda_{\pi\pi} = -\exp(2i\alpha)$. We thus use

$$\sin(2\alpha) = -\text{Im } \lambda_{\pi\pi} \quad (50)$$

⁸In those systems we use for M_{12}^{SM} a simplified expression involving only the exchange of top quarks in the box diagram.

⁹This definition uses the sign conventions in [8]. Many authors use instead $q \rightarrow -q$.

¹⁰The phase ϵ' in equation (BLS-28.24) is known to be tiny.

together with the current experimental value of $\sin(2\alpha)$ to constrain M_{12}^{NP} in the B_d system.

To summarize, we work with 14 parameters: $a, b, c, x, y, a', b', c', x', y', \theta, r = v_2/v_1, m_A$, and m_{eff} . With those 14 parameters we strive to fit 15 observables: $m_u, m_c, m_t, m_d, m_s, m_b, |V_{us}|, |V_{cb}|, |V_{ub}|, \Delta m_K, \epsilon_K, \Delta m_{B_d}, \Delta m_{B_s}, \sin(2\beta)$, and $\sin(2\alpha)$. We found that the fit is possible and, indeed, we have found a large variety of input parameters, *i.e.* of points in parameter space, which are able to satisfy the criteria of the fit.

4.2 Second stage

Each one of the fits found in the previous subsection is *a posteriori* passed through a filter, to ensure that¹¹

the Yukawa couplings are perturbative;

the quantity $\sin(2\beta)$ computed from the decays $B_d \rightarrow D^+ D^-$ is correct;

the angle γ lies in the allowed range;

Δm_D is not too large.

We next explain each of these four points.

From equation (17) we see that the Yukawa-coupling matrix Γ_1 has matrix elements $a/v_1, b/v_1$, and c/v_1 ; likewise, the matrix Γ_2 has elements x/v_2 and y/v_2 . In order to preserve the perturbation expansion, we have required that, for any particular solution in our fit, all matrix elements of Γ_1 and Γ_2 —and, likewise, of Δ_1 and Δ_2 —do not exceed 4π in modulus.

In the decays $B_d \rightarrow D^+ D^-$ one has¹²

$$\lambda_{D^+ D^-} = \left(\frac{q}{p} \right)_{B_d} \frac{V_{cb} V_{cd}^*}{V_{cb}^* V_{cd}} \quad (51)$$

and

$$\sin(2\beta) = \text{Im } \lambda_{D^+ D^-}. \quad (52)$$

¹¹We find that all the fit points which have passed through the filter actually have $|V_{td}|$ in the SM range.

¹²Although there is a loop-suppressed (but not CKM-suppressed) penguin contribution to this decay, this can be ignored due to the large experimental error. It is only the sign of this observable which will be of use below.

We require $\sin(2\beta)$ computed in this way to agree with the experimental value. Notice that this path to $\sin(2\beta)$ does not include M_{12} in the K system.

The CP-violating phase $\gamma = \arg(-V_{ud}V_{cb}V_{ub}^*V_{cd}^*)$ has been extracted from the decays $B^\pm \rightarrow DK^\pm$. There are two experimentally allowed regions: one region in which $\gamma \approx 70^\circ$ is in the first quadrant and has values consistent with the SM, and another region with $\gamma \approx 70^\circ - 180^\circ$ in the third quadrant. The solutions with γ in the third quadrant, though, are excluded by current measurements of the semileptonic asymmetry in B decays [9]. We have computed γ in each of our fit points and used it *a posteriori* in our fit.

The experimental data discussed this far potentially constrain the scalar masses m_A and m_{eff} and the FCNC matrix N_d . The most important constraints on N_u come from mixing (*i.e.* M_{12}) in the D system. In the SM, that mixing has three origins: box diagrams, dipenguin diagrams, and long-distance physics. The long-distance effects should be dominant but are very difficult to estimate reliably. Therefore, we only require that the NP contribution by itself alone should not exceed twice the experimental limit on Δm_D .

We want to comment on a set of points that we have found at the first stage of our fit and which display an inverted unitarity triangle, *i.e.* have a negative Jarlskog invariant [10] $J_{\text{CKM}} = \text{Im}(V_{us}V_{cb}V_{ub}^*V_{cs}^*)$. Such points fit well the 15 observables used in the first stage, but are all eliminated at the second stage of the fit, because they display $\gamma \approx -70^\circ$, in contradiction with experiment. Besides, some of the $J_{\text{CKM}} < 0$ points suffer from the extra problem that they rely on dramatic contributions to M_{12} in the K system, with $M_{12}^{\text{NP}} \approx -2M_{12}^{\text{SM}}$. In these points the sign of q/p in the K system is inverted with respect to the SM. As a result, $\sin(2\beta)$ extracted from ψK_S decays would have the opposite sign to the $\sin(2\beta)$ extracted from D^+D^- decays, which is excluded by experiment.

4.3 Two extra quantities

CP violation has also been measured in the decay $B_s \rightarrow \psi\phi$. It is determined by

$$\lambda_{\psi\phi} = \left(\frac{q}{p}\right)_{B_s} \frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}. \quad (53)$$

Using equation (BLS-30.36), we see that the SM leads to $\lambda_{\psi\phi} = -\exp(2i\beta_s)$, where β_s is a phase in the CKM matrix which, in the SM, is of order a few percent.¹³ Thus, in the SM

$$\sin(2\beta_s) = -\text{Im } \lambda_{\psi\phi}. \quad (54)$$

We might have used the current measurement of $\sin(2\beta_s)$ from the decays $B_s \rightarrow \psi\phi$ to constrain M_{12}^{NP} in the B_s system. However, a recent average [11] excludes the SM at the 2.3σ level. Our fits always yield a β_s very close to its SM value; thus, our model does not provide a solution to this discrepancy between the SM and experiment.

In this model, direct CP violation is negligible in D decays, and therefore [12]

$$\arg\left(\Gamma_{12}^* \frac{\bar{A}_{K^+K^-}}{A_{K^+K^-}}\right) = 0, \quad (55)$$

relates Γ_{12} to the amplitudes for the decays $D \rightarrow K^+K^-$. As a result,

$$\phi_{12} = \arg(M_{12}\Gamma_{12}^*) = -\arg\left(M_{12}^* \frac{V_{cs}V_{us}^*}{V_{cs}^*V_{us}}\right). \quad (56)$$

As shown in reference [12], the theoretical parameter ϕ_{12} can be extracted from the experimental data.

5 The fit: results

After the two stages of our fit we still have many points which have satisfied all the filtering criteria. With those points we have made a number of figures, which we next present.

Figure 1 displays the asymmetry between m_A and m_{eff} as a function of the smallest of those two masses. Clearly, if the scalar masses are both very large, then the model is effectively like the SM, except for the important fact that now the CKM CP-violating phase does not arise from complex hard (dimension-4) Yukawa couplings, as in the SM, but rather from a soft (dimension-2) CP-breaking term in the scalar sector. We find, however, that our model can have scalar masses as small as a few hundred GeV, especially

¹³In [8] the phase β_s has been called ϵ .

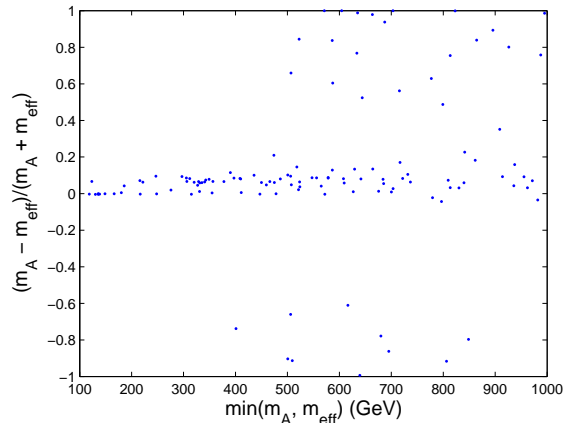


Figure 1: The masses of the scalars. We do not display the points where both m_A and m_{eff} are larger than 1 TeV.

when $m_A \approx m_{\text{eff}}$.¹⁴ In this case of low scalar masses, we have found that $M_{12}^{\text{NP}}/M_{12}^{\text{SM}}$ can be very large in the kaon sector, but is not larger than 10% in the B_d and B_s systems.

In order to quantify the latter statement, we define [9]

$$M_{12} = M_{12}^{\text{SM}} + M_{12}^{\text{NP}} = M_{12}^{\text{SM}} \Delta, \quad (57)$$

where the SM limit corresponds to $\Delta = 1$. We shall use a subscript K, d, s in Δ to refer to the cases of the K system, B_d system, and B_s system, respectively. The current measurements do not agree well with the SM. Setting $\Delta_K = 1$ and excluding the measurement of β_s , the CKMfitter Group [13] finds that the current constraints on Δ_d and Δ_s exclude the SM at the 2.2σ and 1.9σ levels, respectively. The measurements of β_s are much above the SM prediction and further worsen this inconsistency [9]. Similar conclusions are drawn by the UTfit Collaboration [14].

Figures 2, 3, and 4 contain the results of our fits for Δ_K , Δ_d , and Δ_s , respectively. We see that $\text{Im } \Delta$ is in general quite small. This is a reflection of the fact that in our model CP violation lies exclusively in the CKM matrix while the matrix N_d is real; therefore M_{12}^{NP} is, in our model, real in all three

¹⁴Low scalar masses may in some cases be excluded by other experimental constraints that we have not taken into account, for instance by top-quark decays.

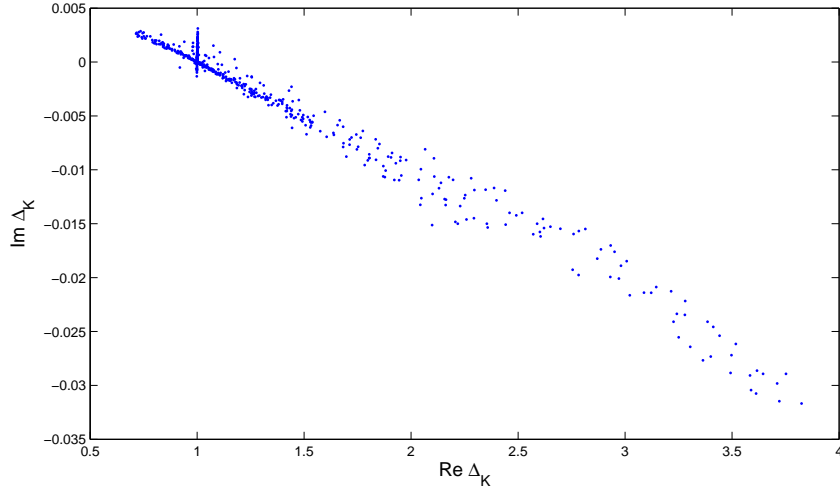


Figure 2: Δ parameter for the K mesons.

neutral-meson systems.¹⁵ We see in Figure 2 that $\text{Re } \Delta_K$ can be as large as three or four. This freedom is due to the large uncertainty in the long-distance contributions to K mixing. On the other hand, since ϵ_K is small, $\text{Im } \Delta_K$ cannot be larger than two or three percent. In the B_d system, changes of Δ_d of order 10% relative to the SM are possible both in the real and imaginary parts. For some of our points this decreases slightly the inconsistency of the SM with the experimental fits. However, this improvement is not dramatic because the experimental fits prefer $\text{Im } \Delta_d < 0$ and $\text{Re } \Delta_d < 1$, while our points with $\text{Im } \Delta_d < 0$ have $\text{Re } \Delta_d > 1$, *cf.* Figure 3. In the B_s system, $\text{Re } \Delta_s$ can differ from 1 by 10% or so, while $\text{Im } \Delta_s$ remains at the 0.1% level. Thus, in the B_s system our model is as (in)consistent with experiment as the SM.

Figure 5 contains the predictions of our model for ϕ_{12} , based on the full set of our points and using exclusively M_{12}^{NP} , *i.e.* assuming $M_{12}^{\text{SM}} = 0$. We see in Figure 5 that $\sin^2 \phi_{12}$ is, in our model, arbitrary; this illustrates how important CP violation in the D system can be in constraining models of new physics [15] such as ours. Notice that the present experimental constraints on ϕ_{12} depend on a set of measurements which are highly correlated [13];

¹⁵We have neglected potentially complex contributions to M_{12}^{NP} at loop level, notably box diagrams involving intermediate charged scalars C^\pm . This is consistent with our previously stated assumption that the NP tree-level contributions to quark decays are much smaller than the SM ones.

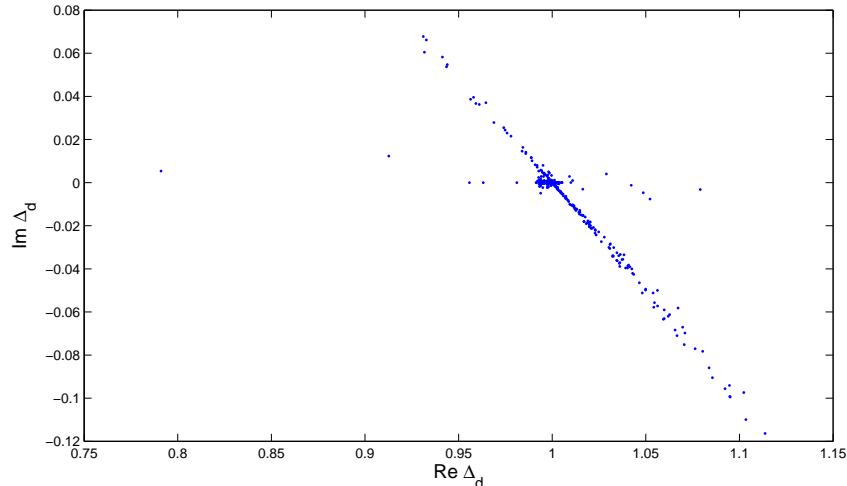


Figure 3: Δ parameter for the B_d mesons.

precise numbers are not available, but we estimate, based on the method in [12], that $\sin^2 \phi_{12} < 0.34$ at the 1σ level.

6 Conclusions

In this paper we have presented a two-Higgs-doublet model with a \mathbb{Z}_3 symmetry and the usual CP symmetry in all the hard (dimension-four) terms but broken in one, and only one, soft (dimension-two) term in the scalar potential. We have shown that this model displays a CP violation which, just as in the SM, is concentrated in the CKM matrix, even though it has a completely different origin. Contrary to most other 2HDMs, our model exhibits CP violation neither in scalar–pseudoscalar mixing, not in the scalar self-interactions, nor in the matrices $N_{d,u}$ which parametrize the flavour-changing Yukawa interactions of the neutral scalars.

Our model has only eleven parameters—ten moduli and one phase—to fit the six quark masses and the four independent observables of the CKM matrix. When computing mixing in the neutral-meson–antimeson systems one needs three extra parameters—the ratio of VEVs, the mass of the pseudoscalar, and a weighted mass of the two scalars. With these parameters one is able to fit most observables, just as in the SM. Remarkably, many of these

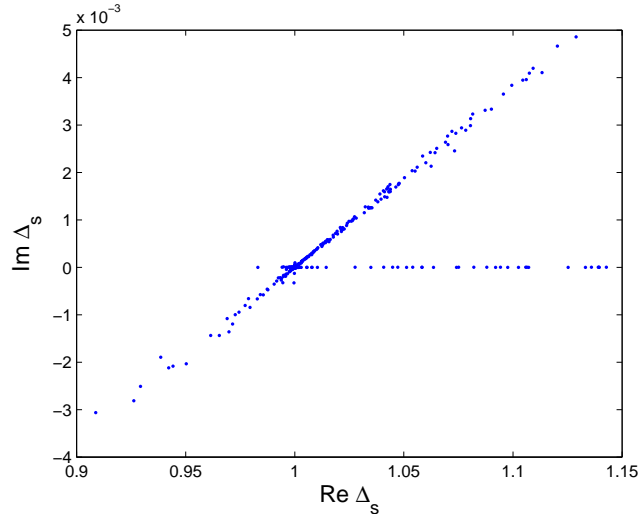


Figure 4: Δ parameter for the B_s mesons.

fits display scalar masses as low as 400 GeV.

We have emphasized the relevance that a measurement of CP violation in D -meson–antimeson mixing may have in eliminating some of our fit points and, thus, in reducing the viable parameter space of our model.

Acknowledgements: The work of L.L. and of J.P.S. is funded by FCT through the projects CERN/FP/109305/2009 and U777-Plurianual, and by the EU RTN project Marie Curie: MRTN-CT-2006-035505. J.P.S. is grateful to Y. Grossman, Y. Nir, M. Papucci, and D. Pirjol for exchanges concerning solutions with negative J_{CKM} .

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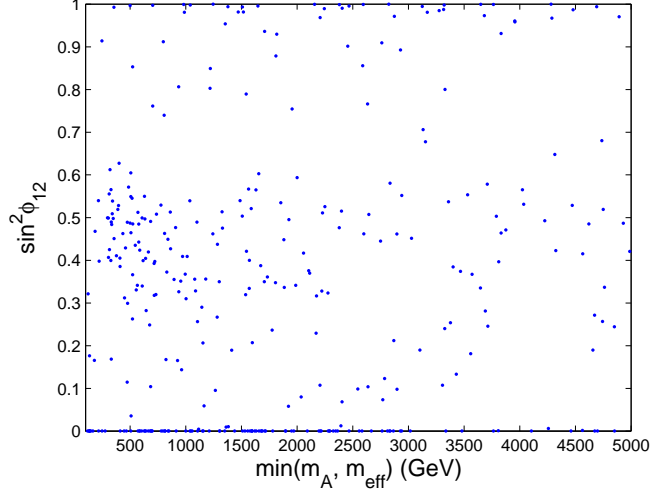


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A Input parameters

We have used in our fits $G_F = 1.16639 \times 10^{-5} \text{ GeV}^{-2}$ and $m_W = 80.4 \text{ GeV}$. In the neutral-kaon system, we have used $m_K = 497.614 \text{ MeV}$, $f_K = 155.5 \text{ MeV}$; for the QCD correction factors of equation (BLS-17.16) we have taken [16] $\eta_1 = 1.38$, $\eta_2 = 0.57$, and $\eta_3 = 0.47$. For the “bag parameter” we have used [13] $B_K = 0.723$. Our results do not depend crucially on these inputs—small variations thereof do not change our conclusions.

In the B_d system, we have used $m_{B_d} = 5.2795 \text{ GeV}$, $f_{B_d} = 190 \text{ MeV}$, $\eta_{B_d} = 0.55$, and $B_{B_d} = 1.219$. For the B_s mesons, [13, 7], $m_{B_s} = 5.366 \text{ GeV}$, $f_{B_s} = 228 \text{ MeV}$, $\eta_{B_s} = 0.55$, and $B_{B_s} = 1.280$. In the D system, $f_D = 232 \text{ MeV}$ [17] and $m_D = 1.86483 \text{ GeV}$ [7].

We next present two of our fit points: one with low masses m_A and m_{eff} and another one in which one of the masses is low and the other one much larger.

- First point: $a = 105 \text{ MeV}$, $b = 15.2 \text{ MeV}$, $c = 6 \text{ MeV}$, $x = 4.1387 \text{ GeV}$, $y = 24.2 \text{ MeV}$, $a' = 169.6184 \text{ GeV}$, $b' = -8.5 \text{ MeV}$, $c' = 1.2823 \text{ GeV}$, $x' = 6.8821 \text{ GeV}$, $y' = -1.8 \text{ MeV}$, $\theta = 5.6548 \text{ rad}$, $r = 0.4277$, $m_A = 415.6327 \text{ GeV}$, and $m_{\text{eff}} = 411.0434 \text{ GeV}$.
- Second point: $a = 17.5 \text{ MeV}$, $b = 180.7 \text{ MeV}$, $c = -27.8 \text{ MeV}$, $x = 4.2504 \text{ GeV}$, $y = 74.9 \text{ MeV}$, $a' = 172.8953 \text{ GeV}$, $b' = -29.6 \text{ MeV}$, $c' = -2.7 \text{ MeV}$, $x' = -600.3 \text{ MeV}$, $y' = 1.2920 \text{ GeV}$, $\theta = 3.6987 \text{ rad}$, $r = 0.8217$, $m_A = 400.9344 \text{ GeV}$, and $m_{\text{eff}} = 2.6596688 \text{ TeV}$.

With these inputs one obtains the following values for the observables:

- First point: $m_d = 5.8 \text{ MeV}$, $m_s = 107.8 \text{ MeV}$, $m_b = 4.1388 \text{ GeV}$, $m_u = 1.8 \text{ MeV}$, $m_c = 1.2813 \text{ GeV}$, $m_t = 169.758 \text{ GeV}$, $|V_{us}| = 0.2256$, $|V_{cb}| = 0.0405$, $|V_{ub}| = 0.0037$, $|V_{td}| = 0.0087$, $\Delta m_K = 2.93 \times 10^{-6} \text{ eV}$, $\Delta m_{B_d} = 3.55 \times 10^{-4} \text{ eV}$, $\Delta m_{B_s} = 1.13 \times 10^{-2} \text{ eV}$, $|\epsilon_K| = 2.227 \times 10^{-3}$,

$J_{\text{CKM}} = 3.113 \times 10^{-5}$, $\sin(2\alpha) = 0.1714$, $\sin(2\beta) = 0.7128$,¹⁶ $\sin(2\beta_s) = -0.0397$, $\gamma = 71.96^\circ$, $\Delta_K = 1.3242 - 0.0032i$, $\Delta_d = 0.9996 + 7.2444 \times 10^{-5}i$, $\Delta_s = 1.0000 - 9.6084 \times 10^{-7}i$.

- Second point: $m_d = 6.0 \text{ MeV}$, $m_s = 81.5 \text{ MeV}$, $m_b = 4.2542 \text{ GeV}$, $m_u = 2.7 \text{ MeV}$, $m_c = 1.2923 \text{ GeV}$, $m_t = 172.8963 \text{ GeV}$, $|V_{us}| = 0.2249$, $|V_{cb}| = 0.0425$, $|V_{ub}| = 0.0037$, $|V_{td}| = 0.0089$, $\Delta m_K = 4.98 \times 10^{-6} \text{ eV}$, $\Delta m_{B_d} = 3.57 \times 10^{-4} \text{ eV}$, $\Delta m_{B_s} = 1.16 \times 10^{-2} \text{ eV}$, $|\epsilon_K| = 2.128 \times 10^{-3}$, $J_{\text{CKM}} = 3.211 \times 10^{-5}$, $\sin(2\alpha) = 0.1167$, $\sin(2\beta) = 0.7450$,¹⁷ $\sin(2\beta_s) = -0.0407$, $\gamma = 69.11^\circ$, $\Delta_K = 2.2183 - 0.0150i$, $\Delta_d = 0.9311 + 0.0678i$, $\Delta_s = 0.9088 - 0.0031i$.

B Oblique parameters

Relevant constraints on the scalar spectrum of a two-Higgs-doublet model arise from consideration of the so-called ‘oblique parameters’, especially of the parameters S and T .¹⁸ Formulae for those parameters in a general MHDM have been presented in ref. [18]. In our particular 2HDM, one has

$$T = \frac{1}{16\pi s_w^2 m_W^2} [\cos^2 \psi f(m_C^2, m_1^2) + \sin^2 \psi f(m_C^2, m_2^2) + f(m_C^2, m_A^2) - \sin^2 \psi f(m_2^2, m_A^2) - \cos^2 \psi f(m_1^2, m_A^2) + \sin^2 \psi f'(m_1^2) + \cos^2 \psi f'(m_2^2) - f'(m_H^2)], \quad (\text{B1})$$

where

$$f(x, y) = \begin{cases} \frac{x+y}{2} - \frac{xy}{x-y} \ln \frac{x}{y} & \Leftarrow x \neq y, \\ 0 & \Leftarrow x = y, \end{cases} \quad (\text{B2})$$

$$f'(m^2) = 3 [f(m_Z^2, m^2) - f(m_W^2, m^2)]. \quad (\text{B3})$$

In equations (B1) and (B3), m_C is the mass of the charged scalars C^\pm , m_W is the mass of the W^\pm , m_Z is the mass of the Z^0 , m_H is the mass of the SM

¹⁶This is the $\sin(2\beta)$ which is obtained from the decays $B_d \rightarrow \psi K_S$. The value obtained from $B_d \rightarrow D^+ D^-$ is 0.7189.

¹⁷The $\sin(2\beta)$ obtained from $B_d \rightarrow D^+ D^-$ is 0.7487.

¹⁸The other oblique parameters are usually very small and, therefore, irrelevant. We have checked this explicitly for some of our points.

Higgs particle, and $s_w^2 = 1 - m_W^2/m_Z^2$. The expression for S is

$$S = \frac{1}{24\pi} \left[(1 - 2s_w^2)^2 g(x_C, x_C) + \sin^2 \psi g(x_2, x_A) + \cos^2 \psi g(x_1, x_A) \right. \\ \left. + \sin^2 \psi \hat{g}(x_1) + \cos^2 \psi \hat{g}(x_2) - \hat{g}(x_H) + \ln \frac{m_1^2 m_2^2 m_A^2}{m_C^4 m_H^2} \right], \quad (\text{B4})$$

where $x_k = m_k^2/m_Z^2$ for $k = 1, 2, A, C$. The functions $g(x, y)$ and $\hat{g}(x)$ in equation (B4) are in the second paper of ref. [18].¹⁹

For each of our fit points we only have the masses m_{eff} —in eq. (41)—and m_A . Equation (41) may be solved for the mixing angle ψ , yielding

$$\sin^2 \psi = \frac{m_1^2}{m_{\text{eff}}^2} \frac{m_2^2 - m_{\text{eff}}^2}{m_2^2 - m_1^2}, \quad \cos^2 \psi = \frac{m_2^2}{m_{\text{eff}}^2} \frac{m_1^2 - m_{\text{eff}}^2}{m_1^2 - m_2^2}. \quad (\text{B5})$$

Therefore, either $m_1 \leq m_{\text{eff}} \leq m_2$ or $m_2 \leq m_{\text{eff}} \leq m_1$.

In order to check whether each of our fit points—defined by given values of m_{eff} and m_A —is compatible with the experimental bounds on the oblique parameters [19], we have inputted various values of $m_{1,2}$ and m_C . From m_1 and m_2 we have computed ψ through equation (B5) and then the oblique parameters S and T . With a fast fitting program we have been able to find, for a large part of our fit points, values of m_1 , m_2 , and m_C such that S and T result compatible with the experimental bounds.²⁰ The masses $m_{1,2,C}$ can be chosen such that the parameter T does not result too large (either positive or negative). The parameter S usually turns out to be positive and relatively large ($S \gtrsim 0.1$), but for most²¹ of our points it can be made compatible with the experimental bounds.

As an example, one of our fit points has $m_{\text{eff}} = 5.4711$ TeV and $m_A = 679.9875$ GeV. Choosing $m_1 = 0.99 m_{\text{eff}}$, $m_2 = 2.0283 m_{\text{eff}}$, and $m_C = 2 m_{\text{eff}}$, one obtains $S = 0.21$ and $T = 0.19$.

¹⁹One has $g(x, y) \equiv G(xz, yz, z)$ and $\hat{g}(x) \equiv \hat{G}(xz, z)$, with the functions $G(I, J, Q)$ in equation (C2) and $\hat{G}(I, Q)$ in equation (C5) of ref. [18].

²⁰We have used $m_H = 117$ GeV in accordance with one of the experimental ellipses in Figure 10.4 of ref [19].

²¹We have *not* been able to explicitly find out, for *all* of our fit points, values of $m_{1,2,C}$ such that both S and T agree with the experimental bounds, but we cannot exclude that that is possible.

C Direct LEP bounds

In Figure 1 we have shown that our fit sometimes yields scalar masses as low as 100 GeV. Since the current limit from LEP is 114.4 GeV [7], it is necessary to verify that our results do not contradict that bound. The LEP result is obtained by looking at the associated production of a scalar particle and a Z^0 boson, $e^+e^- \rightarrow Zh$, which is possible due to the vertex ZZh . For a 2HDM there is also ZH production, but not ZA production, since there is no ZZA vertex. Moreover, as compared to the SM, the coupling of the vertex ZZh (ZZH) is reduced by factors related to the mixing angle ψ in equation (41). Indeed, in our model one has

$$\frac{\sigma^{\text{2HDM}}(e^+e^- \rightarrow ZS)}{\sigma^{\text{SM}}(e^+e^- \rightarrow ZS)} = g_{ZZS}^2, \quad (\text{C1})$$

where $g_{ZZS}^2 = \sin^2 \psi$ ($\cos^2 \psi$) if $S = h$ ($S = H$). Therefore, in a 2HDM it is possible to have scalars with masses lower than the LEP bound, provided those scalars couple more weakly to ZZ than in the SM.

As explained before, our fit to the quark masses, CKM matrix elements, and CP-violating quantities has produced a large number of acceptable points in parameter space. Out of those, as seen in Appendix B, the vast majority conforms to the existing constraints on the oblique parameters. In Figure 6 we plot, for h and H simultaneously, the comparison between the set of points which have passed the oblique-parameter fit and the experimental data from the direct searches at LEP; acceptable points must be below and to the right of the solid line in the plot. We see that, with the exception of only three points, the parameter space that we have found agrees perfectly with the LEP data. (As with the fit to the oblique parameters, we cannot exclude that other values of $m_{1,2,C}$ can be found, such that all the points agree with the LEP experimental bounds.)

We have also looked at the existing LEP bounds on scalar–pseudoscalar production. Those bounds extend to 225 GeV in the sum of the masses of the scalar and the pseudoscalar. We have found that all our points which survive the LEP bounds on Z^0 –scalar production display a sum of the masses of the scalar and the pseudoscalar which exceeds 225 GeV. Therefore, all those points also survive the LEP bounds on scalar–pseudoscalar production.

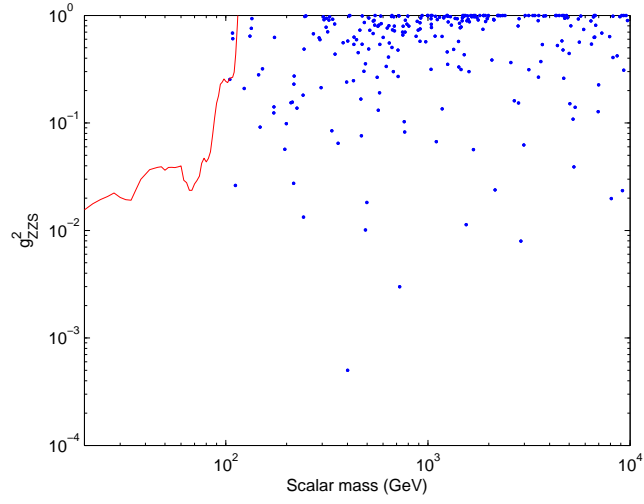


Figure 6: Constraints on scalar masses from direct searches at LEP (data taken from ref. [20]).